

KENDRIYA VIDYALAYA SANGATHAN

12th KVS Junior Mathematics Olympiad - 2009

Time : 3 Hours

M.M. 100

NOTE: Attempt all questions. All questions carry equal marks.

The use of electronic devices are strictly prohibited.

Q. No. 1 Consider the following multiplication in decimal notations

$(999).(abc) = def132$, determine the digits a,b,c,d,e,f.

Q. No. 2 Find the greatest number of 4 digits , which when divided

by 3,5,7, and 9 leaves remainder 1,3,5 and 7 respectively.

Q. No.3 If n is a positive integer such that $\frac{n}{810} = 0.d25d25\dots$

where d is a single digit in decimal base. Find 'n'.

Q. No.4 Solve in integers:

$$3x^2 - 3xy + y^2 = 7 \quad \text{and} \quad 2x^2 - 3xy + 2y^2 = 14$$

Q. No.5 Let x be the LCM of $3^{2002} - 1$ and $3^{2002} + 1$. Find the last digit of x.

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Q. No.6 Let $f_0(x) = \frac{1}{1-x}$ and $f_n(x) = f_0(f_{n-1}(x))$

Where $n=1,2,3,\dots$. Calculate $f_{2009}(2009)$

Q. No.7 $\triangle ABC$ and $\triangle DAC$ are two isosceles triangles with $\angle BAC = 20^\circ$ and $\angle ADC = 100^\circ$. Show that $AB = BC + CD$.

Q. No. 8 Two intersecting circles E_1 and E_2 have a common tangent which touches E_1 at P and E_2 at Q. These two circles meet at M and N where N is nearer to PQ than M. The line PN meets the circle E_2 again at R. Prove that MQ bisects $\angle PMR$.

Q. No. 9 AB is a line segment of length 24 cm. and C is its middle point. On AB, AC and CB semi circles are described. Determine the radius of the circle which touches all the three semi circles.

Q. No.10 Prove that $a^4 + b^4 + c^4 \geq abc(a + b + c)$