# RMO–1992

1. Determine the set of integers $n$ for which $n^2 + 19n + 92$ is a square of an integer.
2. If $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$, where $a$, $b$, $c$ are positive integers with no common factor, prove that $(a + b)$ is the square of an integer.
3. Determine the largest 3-digit prime factor of the integer ${}^{2000}C_{1000}$.
4. $ABCD$ is a cyclic quadrilateral with $AC \perp BD$; $AC$ meets $BD$ at $E$. Prove that

$$EA^2 + EB^2 + EC^2 + ED^2 = 4R^2,$$

where $R$ is the radius of the circumscribing circle.

5. $ABCD$ is a cyclic quadrilateral; $x$, $y$, $z$ are the distances of $A$ from the lines $BD$, $BC$, $CD$ respectively. Prove that

$$\frac{BD}{x} = \frac{BC}{y} + \frac{CD}{z}.$$

6. $ABCD$ is a quadrilateral and $P$, $Q$ are mid-points of $CD$, $AB$ respectively. Let $AP$, $DQ$ meet at $X$, and $BP$, $CQ$ meet at $Y$. Prove that

area of $ADX$ + area of $BCY$ = area of quadrilateral $PXQY$.

7. Prove that

$$1 < \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + \ldots + \frac{1}{3001} < 1\frac{1}{3}.$$

8. Solve the system

$$\begin{aligned} (x+y)(x+y+z) &= 18 \\ (y+z)(x+y+z) &= 30 \\ (z+x)(x+y+z) &= 2A \end{aligned}$$

in terms of the parameter $A$.

9. The cyclic octagon $ABCDEFGH$ has sides $a$, $a$, $a$, $a$, $b$, $b$, $b$, $b$ respectively. Find the radius of the circle that circumscribes $ABCDEFGH$ in terms of $a$ and $b$.