RMO–1999

1. Prove that the inradius of a right-angled triangle with integer sides is an integer.
2. Find the number of positive integers which divide 10^{999} but not 10^{998}.
3. Let $ABCD$ be a square and M, N points on sides AB, BC, respectably, such that $\angle MDN = 45^\circ$. If R is the midpoint of MN show that $RP = RQ$ where P, Q are the points of intersection of AC with the lines MD, ND.
4. If p, q, r are the roots of the cubic equation $x^3 - 3px^2 + 3q^2x - r^3 = 0$, show that $p = q = r$.
5. If a, b, c are the sides of a triangle prove the following inequality:
$$\frac{a}{c+a-b} + \frac{b}{a+b-c} + \frac{c}{b+c-a} \geq 3.$$
6. Find all solutions in integers m, n of the equation
$$(m-n)^2 = \frac{4mn}{m+n-1.}$$
7. Find the number of quadratic polynomials, $ax^2 + bx + c$, which satisfy the following conditions:
   (a) a, b, c are distinct;
   (b) $a, b, c \in \{1, 2, 3, \ldots 1999\}$ and
   (c) $x + 1$ divides $ax^2 + bx + c$.