RMO–1990

1. Two boxes contain between them 65 balls of several different sizes. Each ball is white, black, red or yellow. If you take any 5 balls of the same colour at least two of them will always be of the same size (radius). Prove that there are at least 3 balls which lie in the same box have the same colour and have the same size (radius).
2. For all positive real numbers a, b, c prove that
$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}.$$
3. A square sheet of paper $ABCD$ is so folded that B falls on the mid-point M of CD. Prove that the crease will divide BC in the ratio $5 : 3$.
4. Find the remainder when 2^{1990} is divided by 1990.
5. P is any point inside a triangle ABC. The perimeter of the triangle $AB + BC + CA = 2s$. Prove that
$$s < AP + BP + CP < 2s.$$
6. N is a 50 digit number (in the decimal scale). All digits except the 26th digit (from the left) are 1. If N is divisible by 13, find the 26th digit.
7. A censusman on duty visited a house which the lady inmates declined to reveal their individual ages, but said — "we do not mind giving you the sum of the ages of any two ladies you may choose". Thereupon the censusman said — "In that case please give me the sum of the ages of every possible pair of you". The gave the sums as follows : $30, 33, 41, 58, 66, 69$. The censusman took these figures and happily went away. How did he calculate the individual ages of the ladies from these figures.
8. If the circumcenter and centroid of a triangle coincide, prove that the triangle must be equilateral.