# RMO–2005

1. Let $ABCD$ be a convex quadrilateral; $P, Q, R, S$ be the midpoints of $AB, BC, CD, DA$ respectively such that triangles $AQR$ and $CSP$ are equilateral. Prove that $ABCD$ is a rhombus. Determine its angles.

2. If $x, y$ are integers and 17 divides both the expressions $x^2 - 2xy + y^2 - 5x + 7y$ and $x^2 - 3xy + 2y^2 + x - y$, then prove that 17 divides $xy - 12x + 15y$.

3. If $a, b, c$ are three real numbers such that $|a - b| \geq c$, $|b - c| \geq a$, $|c - a| \geq b$, then prove that one of $a, b, c$ is the sum of the other two.

4. Find the number of all 5-digit numbers (in base 10) each of which contains the block 15 and is divisible by 15. (For example, 31545, 34155 are two such numbers.)

5. In triangle $ABC$, let $D$ be the midpoint of $BC$. If $\angle ADB = 45^\circ$ and $\angle ACD = 30^\circ$, determine $\angle BAD$.

6. Determine all triples $(a, b, c)$ of positive integers such that $a \leq b \leq c$ and
$$a + b + c + ab + bc + ca = abc + 1.$$

7. Let $a, b, c$ be three positive real numbers such that $a + b + c = 1$. Let
$$\lambda = \min\{a^3 + a^2bc, b^3 + ab^2c, c^3 + abc^2\}.$$
Prove that the roots of the equation $x^2 + x + 4\lambda = 0$ are real.