Problem of the week

There are k candidates (where k is an odd number) for the Bookie Prize. The rules of the Bookie Prize say that at least 1 but not more than 50% of the candidates must be awarded a prize. If the number of possible ways of selecting the awardees is 63, find k.

Solution:

Explanation:

We know that k is odd. Let k be 2n+1. Then at least 1 and at most n candidates can be awarded the prize. Hence ${}^kC_1 + {}^kC_2 + {}^kC_3 + \dots + {}^kC_n = 63$. Adding ${}^kC_0 = 1$ to both sides,

${}^kC_0 + {}^kC_1 + {}^kC_2 + {}^kC_3 + \dots + {}^kC_n = 64$

Now ${}^kC_0 = {}^kC_k$ (i.e. ${}^kC_{2n+1}$) and so on hence we can say ${}^kC_0 + {}^kC_1 + {}^kC_2 + {}^kC_3 + \dots + {}^kC_k = 64 + 64 = 128$ i.e. $2^k = 128$ i.e. $k = 7$.

Alternatively,

if $n = 1$, $k = 3$

$\therefore {}^kC_0 + {}^kC_1 + \dots + {}^kC_n = {}^3C_0 + {}^3C_1 = 1 + 3 = 4 = 2^2$

If $n = 2$, $k = 5$

$\therefore {}^kC_0 + {}^kC_1 + \dots + {}^kC_n = {}^5C_0 + {}^5C_1 + {}^5C_2 = 1 + 5 + 10 = 16 = 2^4$

From pattern, if $n = 3$, $k = 7$ and ${}^kC_0 + {}^kC_1 + \dots + {}^kC_1 = 2^6 = 64$