There are eight points in the plane such that no three of them are collinear. Find the maximum number of triangles formed out of these points such that no two triangles have more than one vertex in common.

Solution:

First we observe that each vertex can be present in at most 3 triangles, for, having chosen a vertex, there remains 7 points from which 3 pairs are possible. If there are 9 or more triangles, these account for at least 27 vertices, repetitions allowed. In that case, one vertex has to occur in at least 4 triangles, a contradiction. Thus, there can be at most 8 triangles. The following example shows that 8 are possible; name the points as 1,2,3,4,5,6,7,8; the triangles are with vertices 123, 145, 167, 248, 368, 578, 256, 347.