

Problem of the week

4 spheres of radius 1 are placed so that each touches the other three. What is the radius of the smallest sphere that contains all 4 spheres?

Solu:

The centers of the 4 spheres form a regular tetrahedron side 2. The center of the fifth sphere must obviously be at the center of the tetrahedron. An altitude of the tetrahedron will have one end at the centroid of a face, so its length is

$$\sqrt{2^2 - \left(\frac{2}{3}\sqrt{3}\right)^2} = \sqrt{4 - \frac{4}{3}} = 2\sqrt{\frac{2}{3}}.$$

So the distance from the center of the fifth sphere to one of the other centers is

$$\left(\frac{3}{4}\right)2\sqrt{\frac{2}{3}} = \sqrt{\frac{3}{2}}.$$

Hence the radius of the fifth sphere is

$$\sqrt{\frac{3}{2}} + 1$$