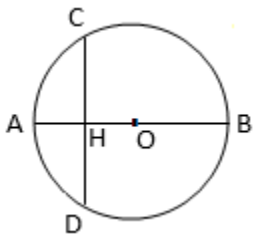
Kendriya Vidyalaya Sangathan, New Delhi

19th KVS Junior Mathematical Olympiad- 2016

M.M. -100 Time : 3 hrs

All questions are compulsory. Each question carry 10 marks.

1 The length of diameter AB is a two digit integer. Reversing the digits gives the length of a perpendicular chord CD. The distance from their intersection point H to the center O is a positive rational number. Determine the length of AB.



2 In Δ ABC, the length of sides AB, BC and CA are 13 cm, 14cm and 15cm respectively. Let 'H' be the orthocenter of the ΔABC and AD is perpendicular is BC. The distance from H to the center of the circle circumscribing ΔABC may be written in simplified form as $\frac{\sqrt{p}}{q}$ where p and q are positive integers.

Determine the value of p+q.

3. Find all integer solutions of the system: $35x+63y+45z=1$, $|x|<9$, $|y|<5$, $|z|<7$,

4. Let ABCD be a parallelogram. Two points E and F are chosen on the sides BC and CD, respectively, such that $\frac{EB}{EC}$ =m, and $\frac{FC}{FD}$ = n, Lines AE and BF interest at G. Prove that the ratio

$$\frac{AG}{GE} = \frac{(m+1)(n+1)}{mn}$$

5. (a) Assume that a,b,c and d are positive integers such that $a^5=b^4$, $c^3=d^2$ and c-a=19 Determine d-b

(b) Find all the pairs (x,y), x>y ≥2, of integers, such that $x^y=y^{x-y}$

6. Find the least positive real number K such that for any positive real numbers x,y,z, the following inequality holds.

$$x\sqrt{y} + y\sqrt{z} + z\sqrt{x} \leq K\sqrt{(x+y)(y+z)(z+x)}$$

7. Let a,b,c be the sides opposite the angles A, B and C respectively of a ΔABC.

Find the value of k such that

(a) a+b=kc (b) $Cot\frac{A}{2}+Cot\frac{B}{2}=k\,Cot\frac{C}{2}$

8. Consider the collection of all 5-digit numbers such that the sum of digits of each number being 43. A number is selected at random from the collection. Find the probability that the number is divisible by 11.

9. (a) If $x^5-x^3+x=a$, $x>0$. Prove that $x^6 \geq 2a-1$.

(b) Let f(x) be a quadrate polynomial with integer coefficients such that f(0) and f(1) are odd integers. Prove that the equation f(x)=0 does not have an integer solution.

10. Evaluate $[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\cdots+\frac{1}{\sqrt{10000}}]$, where [x] is greatest integer $\leq$ x.