

Kendriya Vidyalaya Sangathan, New Delhi
18th KVS Junior Mathematical Olympiad- 2015

Time : 3 hrs

M.M. -100

NOTE: All questions are compulsory, Each question carry 10 marks.

1. How many integers from 1 to 1000, do not have a common factor(s) with 1000?
2. (a) Find the sum of all the digits of the result of $10^{99}-99$.
(b) When 31513 and 34369 are divided by a certain three digit number, the remainders are equal, Find the remainder.
3. In ΔPQR , show that
$$\sqrt{\cos \frac{P}{2}} \sqrt{\cos \frac{Q}{2}} \sqrt{\cos \frac{R}{2}} \geq \frac{1}{2\sqrt{3}} (\sqrt{\sin P} + \sqrt{\sin Q} + \sqrt{\sin R})$$
4. One root of the equation $x^4-5x^3 + ax^2 +bx+c=0$ is $3+\sqrt{2}$. If all the roots of the equation are real find the extreme (maximum or minimum) values of a,b and c and roots of the equation for these extreme values of a,b and c, it is given that a,b and c are rational.
5. ABC is a right angled triangle with $\angle C=90^\circ$. The centre and the radius of the inscribed circle are O and r respectively. Show that
$$AO \times BO = \sqrt{2} \times AB \times r$$
6. (a) ABCD is a square A is joined to the point X in DC and D to the point Y in CB, so that $AX=DY$. Prove that $AX \perp DY$.
(b) In a triangle, the length of an altitude is 4cm and this altitude divides the opposite side in two parts in the ratio 1:8, Find the length of a segment parallel to the altitude which bisects that area of the given triangle.
7. A polynomial $f(x)$ with rational coefficients leaves remainder 15, when divided by $x-3$ and remainder $2x+1$, when divided by $(x-1)^2$. Find the remainder when $f(x)$ is divided by $(x-3)(x-1)^2$.
8. An odd number of sticks lie along a straight path in a particular type of game, the distance between consecutive sticks being 10 meters. The sticks are to be collected at the place where the middle sticks lies. A player can pick only one stick at a time. He has to start picking the sticks beginning from the extreme stick of one side. If total distance covered by the player is 3 km, find how many sticks would have been used in the game.
9. (a) A square of 10 cm side has a circle of 1 cm diameter, whose centre is the centre of the square. A coin of diameter 1 cm is tossed onto the square. Find the probability that this tossed coin intersects the circle.
(b) You have n loads, each of weight w. When they are weighed in pairs, the sum of the weights of all the possible pairs is 120. When they are weighed in triplets, the sum of the weights of all the possible triplets is 480 Find n.

10. Let ABC be an acute angled triangle with $AC \neq BC$ and let O be the circumcenter and F the foot of altitude through C . Further, let X and Y be the feet of perpendiculars dropped from A and B respectively to (the extension of) CO . The line FO intersects the circumcircle of $\triangle FXY$, second time at P . Prove that $OP < OF$