INMO–1997

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1. Let $ABCD$ be a parallelogram. Suppose a line passing through C and lying outside the parallelogram meets AB and AD produced at E and F respectively. Show that

$$AC^2 + CE \cdot CF = AB \cdot AE + AD \cdot AF.$$

2. Show that there do not exist positive integers m and n such that

$$\frac{m}{n} + \frac{m+1}{n} = 4.$$

3. If a, b, c are three distinct real numbers and

$$a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a} = t$$

   for some real number t, prove that $abc + t = 0$.

4. In a unit square one hundred segments are drawn from the center to to the sides dividing the square into one hundred parts (triangles and possibly quadrilaterals). If all the parts have equal perimeter p show that $1 \cdot 4 < p < 1 \cdot 5$.

5. Find the number of 4×4 arrays whose entries are from the set $\{0, 1, 2, 3\}$ and which are such that the sum of numbers in each of the four rows and each of the four columns is divisible by 4. (An $m \times n$ array is an arrangement of mn numbers in m rows and n columns.)

6. Suppose a and b are two positive real numbers such that the roots of the cubic equation

$$x^3 - ax + b = 0$$

   are all real. If α is a root of this cubic with minimum absolute value, prove that

$$\frac{b}{a} < \alpha \leq \frac{3b}{2a}.$$